Sage Model Notes

AcousticStirlingSwiftBackhaus.scfn

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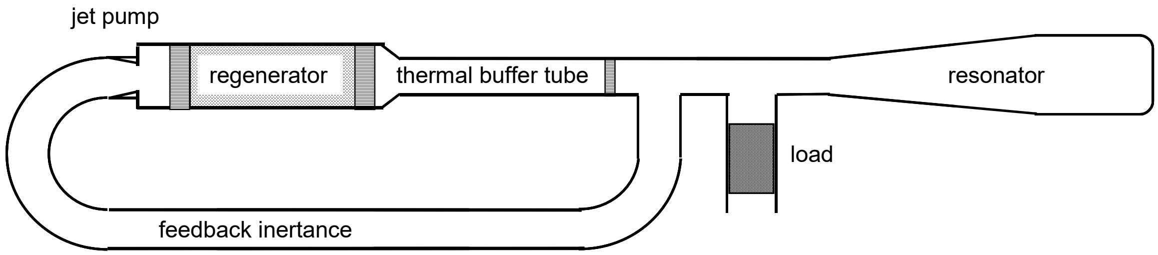
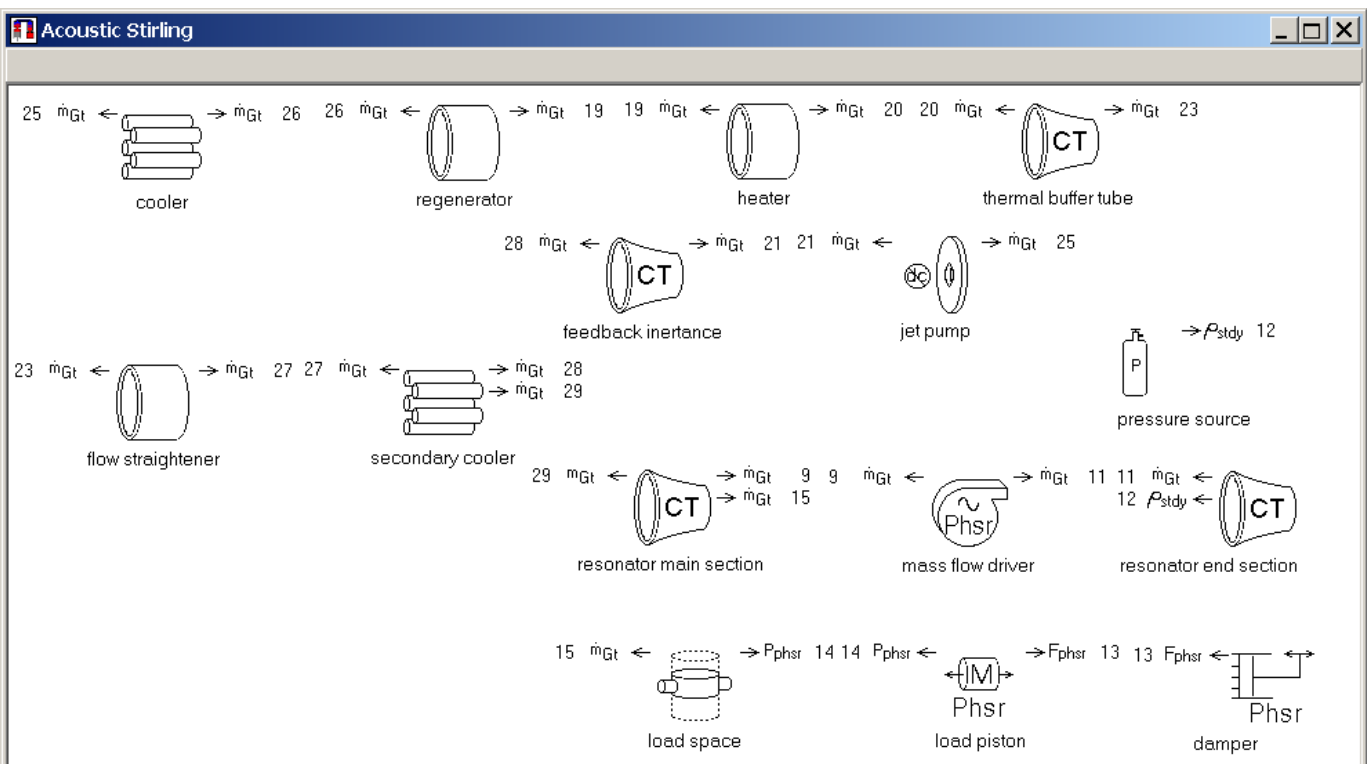
A model of the acoustic stirling engine built and tested at Los Alamos National Laboratory[1, 2]. Here is a schematic of the engine layout:



In the Sage model all the components arranged in the top-level edit window:



1 S. Backhaus and G.W. Swift, *A thermoacoustic-stirling heat engine*, Los Alamos report LA-UR#99-306, (1999)

2 S. Backhaus and G.W. Swift, *A thermoacoustic-stirling heat engine: detailed study*, Los Alamos report LA-UR#99-2283, (1999)

Mass-Flow Driver

A key component of the model is the *mass-flow driver*. Essentially it is a numerical necessity not actually present in the real hardware, but without which the solution would not converge. One thing it does is establish the reference phase for the solution.

In most Sage stirling engine models there is a piston-type compressor driving the solution. The motion of this piston, amplitude and phase, is specified as input. The rest of the solution followed as a consequence. Without a constrained piston, the solution has no phase reference. There would be infinitely many valid solutions, shifted in time.

The mass flow driver accomplishes the same sort of thing in the acoustic stirling model as the constrained piston does in a hard-piston model. Only without the need for a piston. Think of it as a special sort of orifice through which gas flows. What makes it special is that instead of imposing a pressure-drop flow resistance it provides a phasor pressure drop (or rise) in order to bring the first harmonic of the mass flow rate to a setpoint value. It allows any dc component of mass flow rate and any higher harmonics to pass through unobstructed.

What about that artificial pressure-drop phasor imposed by the *mass flow driver*? Physically it is nonsense but the optimizer is set up to solve for the operating frequency and damping coefficient on the load piston in order to zero both components of the pressure-drop phasor. When the pressure-drop phasor is zero, the *mass flow driver* does not affect the solution in any way. This mimics what the real machine does when you adjust the acoustic load to achieve a desired operating point.

It would likely be possible to remove the *mass flow driver* and instead impose the solution phase reference by replacing the free *load piston* with a constrained piston. In effect wagging the dog with the tail. The issue of finding the correct operating frequency would remain only instead of zeroing the pressure drop across the *mass flow driver* you would adjust frequency and piston amplitude (or damping coefficient if the attached damper remains in the model) so that the real and imaginary parts of the required external forcing function on the constrained piston (output *FF*) were zero, (e.g. *FF.Real* = 0 and *FF.Imag* = 0).

Feedback Inertance

The main new thing about the lumped-element acoustic stirling engine compared to earlier thermoacoustic technology is what Backhaus and Swift call the *feedback inertance*. Its role is a lot like the mechanical displacer in a free-displacer stirling engine

In the Sage model, the *feedback inertance* component also includes the U-bend compliance at the left of the schematic. The whole feedback inertance is modeled as a tapered compliance tube with the length being, roughly, the total center-line arc length from the secondary cooler exit to the jet pump. The shape profile is defined by using 6 diameter interpolation points based on diameters for the various sections. To resolve all this in the computational grid, the model uses 12 control volumes along its length (*NCell* = 12).

Jet Pump

The *jet pump* is what controls the dc flow in the stirling engine loop opened up by the addition of the displacer duct. In reality, it is a couple of tapered passages. It could be modeled using the *asymmetric sharp-edged orifice* component in Sage, but this would require constant fiddling with the recovery area ratio to block dc flow. So, instead, it is modeled with a generic dc flow blocking orifice, which allows you to specify any desired amount of dc flow as a set point without regard for the hardware contortions actually required to make it so.

The DC flow blocking orifice includes the pressure drop associated with flow through a simple symmetric orifice of diameter specified by the input variable *Dorf.* In order to capture the time-average pumping dissipation in the actual jet pump, the orifice area is set to 2.18 cm^2 (*Dorf* = 1.67 cm). The relationship between the mean-effective area A of a symmetric orifice and the negative and positive discharge areas of the jet pump A_+ and A_- is

$$\frac{1}{A^2} = \frac{1}{2}\left(\frac{1}{A_+^2} + \frac{1}{A_-^2}\right)$$

The easiest way to see this is to note that for a given volumetric flow rate $\dot{V}$, the jet pump absolute pressure drop in the positive direction is $\rho\dot{V}^2/2A_+^2$ and for the negative direction is $\rho\dot{V}^2/2A_-^2$. So the average absolute pressure drop is $1/4\,\rho\dot{V}^2\left(1/A_+^2 + 1/A_-^2\right)$. On the other hand for a simple orifice the absolute pressure drop regardless of direction is $\rho\dot{V}^2/2A^2$. Canceling out $\rho\dot{V}^2/2$ gives the above area relationship. Substituting 3.0 cm^2 for A_+ and 1.8 cm^2 for A_- gives A = 2.18 cm^2.

Load

The load is a variable-volume space with attached free-piston, branching off the resonator between the main section and end section. The free-piston has low mass (1 gm) and relatively large frontal area (20 cm^2) so it transmits most of its pressure force, except for inertia, to a damper, where the actual power is absorbed. The bottom-line acoustic PV power delivered to the load (user variable *Wload* is taken to be the power absorbed by the damper (*Wpowerpiston*) less any extra PV power produced by the mass flow driver (*Wdriver*). The reason for subtracting *Wdriver* from *Wpowerpiston* is because the difference should be relatively constant, even if the optimizer has not yet driven *Wdriver* to zero.

Displacer Duct Force Balance

The displacer duct is analogous to the physical free-displacer in a free piston stirling engine. A physical free-displacer is a thin-walled bottle-structure which is generally supported by some sort of spring and also has a drive rod, or other differential area, acted upon by the cycle pressure amplitude. Generally speaking, the spring supports the displacer inertia and the drive-rod provides power input necessary to overcome the flow resistance in the regenerator. More formally, the displacer mass times acceleration must equal the sum of forces (gas, spring and drive-rod) acting upon it, according to Newton's second law. When restricted to first harmonics only, Newton's law may be represented in a phasor diagram where the length of the phasor represents the magnitude of the force and the counterclockwise angle with respect to the positive x-axis the phase.

In the case of the acoustic displacer duct, there are two important differences compared to a conventional free displacer: It has compliance and lacks a drive rod. The compliance makes it a bit confusing to pin down exactly what mass we are talking about in applying Newton's law. The obvious first approximation is to take the mass flow rate at the displacer duct midpoint as representative. The lack of a drive rod makes it somewhat unclear what force is *driving* displacer motion. The driving force must arise from the pressure difference at the two ends of the displacer duct. The only source of this pressure difference, other than the displacer duct itself, is the leg containing the regenerator, heat exchangers and thermal buffer tube. Within these components the dominant pressure drop is that due to flow resistance in the regenerator. So, in this sense, the regenerator pressure drop is the most important force, driving the displacer.

The *rodless displacer* is not entirely new to the world. Rodless (though mechanical) free-displacer stirling cryocoolers have been around for some time now. Also there was a variation of the stirling engine known as the *fluidyne* that was somewhat fashionable in the 1970's. The fluidyne used water-filled U-tubes as pistons and displacers. Its purpose was to pump water. But specific power was low due to operating frequency around one Hz and using atmospheric air as the working fluid. The Los Alamos acoustic stirling seems to have remedied both these deficiencies.

Power Flow

Something not done in conventional stirling engine practice is to take the power output directly out of the expansion-space, as it is done in the Los-Alamos engine. The reason this is not done is that it would require a piston operating in the hot part of the engine. Either that or a temperature isolation duct (thermal buffer tube) as in the Los Alamos Design. Rather than doing that, conventional stirling engines always take the power out of a piston operating on the compression-space side. Arguably, this is less efficient because it places an even greater burden on the displacer to feedback power from the expansion space to the compression space than it otherwise would. On the other hand, rigid displacers are very good about transmitting power without losses.

The fact that power is produced in the expansion space and absorbed in the compression space is a consequence of the P-V phasing required in the regenerator, which is essentially the in-phase relationship of a traveling acoustic wave, traveling in the direction from the compression-space (cold) end to the expansion-space (hot) end.